The Kissing Problem

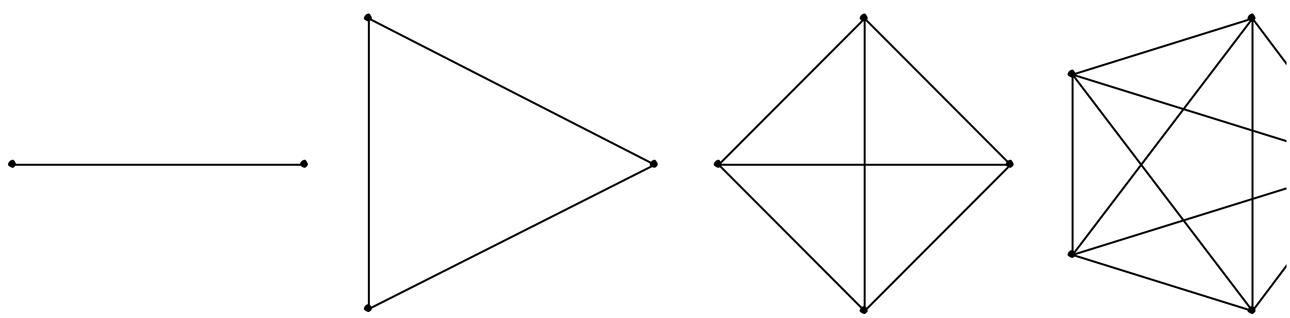
Contributed by Bill Harbaugh, with advice from Anna Harbaugh.

When I went on sabbatical to Aix-en-Provence, my daughter went to french school. She learned that school is a little different in France. For one thing, every day before class, everyone in each of Anna's classes must kiss each other. We are not concerned with why they do this, it is is just the way it is in France. But we would like to know how many kisses it takes. For simplicity, we will ignore the fact that people can kiss each other from two times (normal, for Provence) up to as many as four or even five times (really, really good friends in Provence, but nothing special in Paris) and just define one kiss as occuring when two people kiss each other, any number of times.

One way to think about this problem is to imagine that everyone is arranged in a circle. Put a dot on the circle for each person. Draw a line from each person on the circle to every other person on the circle. Each line represents a kiss. Because of the way we define a kiss, there is only one line between any two people, not one line going each way.

We can display groups of people and kisses this way using Mathematica, as shown below for groups of people from size two to five.

```
In[1]:= << DiscreteMath`Combinatorica`
        ShowGraphArray[Table[CompleteGraph[n], {n, 2, 5}]]
```



```
Out[2]= - GraphicsArray -
```

Now, we would like to know how many kisses it takes until everyone in the class has kissed. Instead of drawing the picture and counting the lines, we will use mathematics to think about the problem and find a general solution, that is a solution that works for any number of people. First, we'll look at the figure for five people, as an example.

Pick one of the dots. Notice that this person must kiss 4 other people - one less than the total number of f 5. And every other person must also kiss 4 others. So, is the total number of kisses 5x4=20? No, because once one person has kissed a person, that person doesn't need to kiss them back, at least according to the way we've defined a kiss. So, one kiss counts for two people, or one line between two dots represents one kiss between two people, and the total number of kisses needed for five people is (5x4)/2=10.

Suppose there are some other number of people, say n people? Following the logic from above, the rule is just (n(n-1))/2 kisses. We can show this in a list, like this, where the left hand column shows the number of people, and the right hand shows the number of kisses. We go up to 35 people because the law in France is that there can't be more than 35 kids in a class.

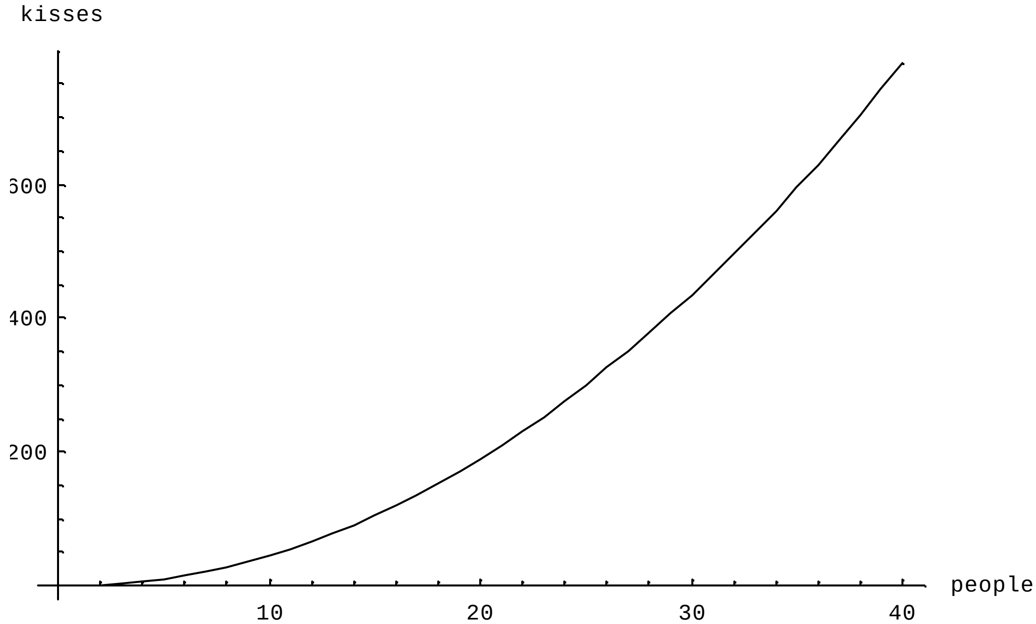
```
In[3]:= TableForm[Table[{n, Binomial[n, 2]}, {n, 1, 35}]]
```

Out[3]//TableForm=

1	0
2	1
3	3
4	6
5	10
6	15
7	21
8	28
9	36
10	45
11	55
12	66
13	78
14	91
15	105
16	120
17	136
18	153
19	171
20	190
21	210
22	231
23	253
24	276
25	300
26	325
27	351
28	378
29	406
30	435
31	465
32	496
33	528
34	561
35	595

Or we can plot it, like this.

```
In[4]:= ListPlot[Table[{n, Binomial[n, 2]}, {n, 1, 40}],
         PlotJoined -> True, AxesLabel → {people, kisses}]
```

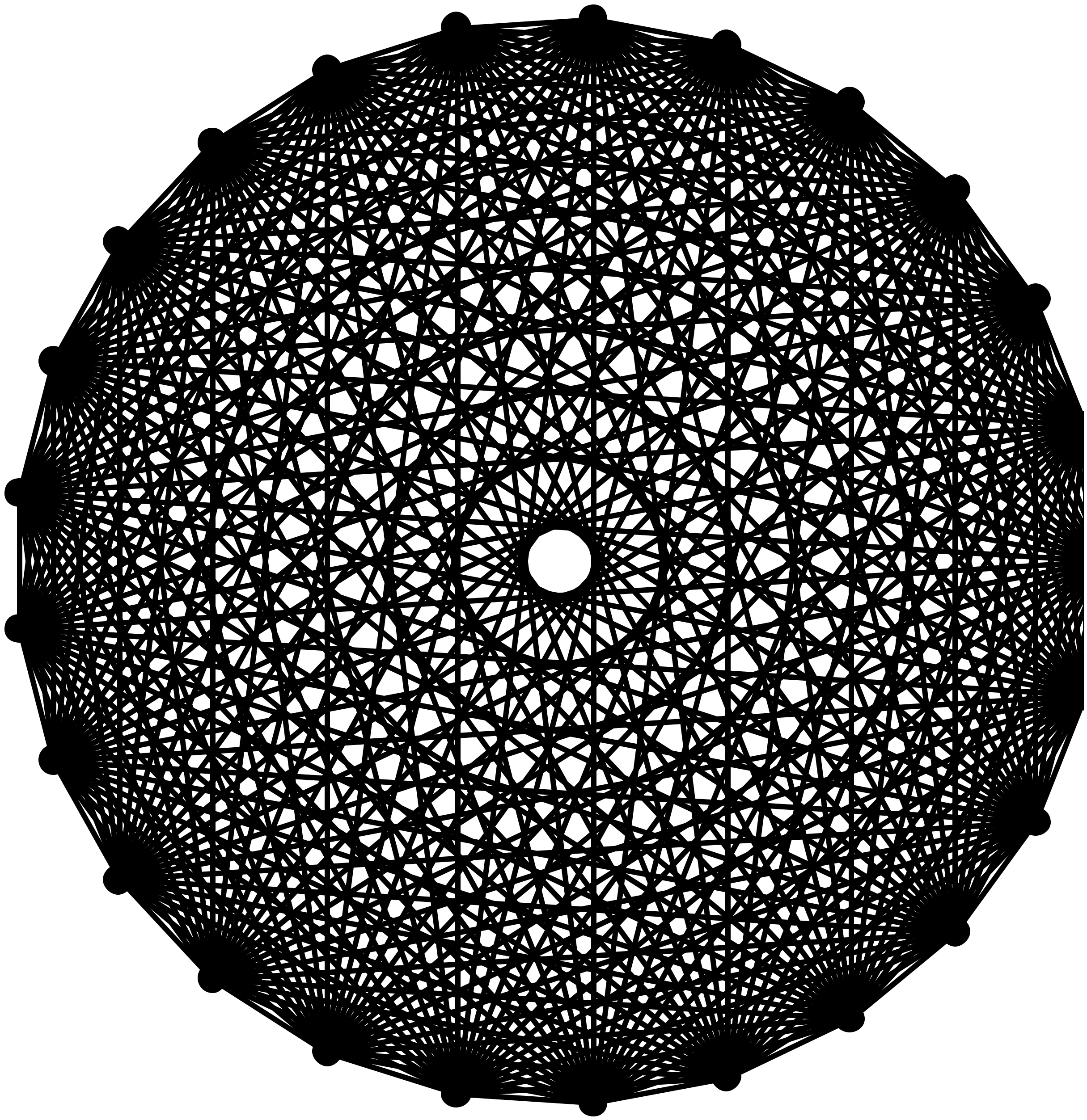



```
Out[4]= - Graphics -
```

So, the number of kisses increases at an increasing rate. With 35 people, Anna's class must engage in a total of 595 kisses. This is a lot of kissing. Maybe that's why they don't want more than 35 kids in a class?

The figure below show the kissing for just 25 people.

```
In[5]:= n = 25;
        ShowGraph[CompleteGraph[n]]
```



```
Out[6]= - Graphics -
```

One thing we haven't dealt with is how to accomplish this kissing in the most efficient manner. Since schools in France only have 10 minutes between classes, efficiency in kissing is of the utmost importance to an orderly educational process. Suppose that it takes 5 seconds for a kiss. How quickly can a class of 35 people kiss each other? How should the kissing be organized?